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On G–transitive version of perfectly meager sets

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Abstract. We study the $G-$ invariant version of perfectly meager sets (a generalization of the notion of AFC$'$ sets). We find the necessary and sufficient conditions for the inclusion $\mathrm{AFC}'_G \subseteq \mathcal{I}$. In particular, we partially characterize for which groups G of automorphisms of the Cantor space every AFC'_G set is Lebesgue null.

1. Definitions and notation

We consider the Cantor space 2^ω as a topological group (where $(x+y)(k) = x(k)+y(k) \mod 2$). By $2^{<\omega}$ let us denote the collection of all finite binary sequences: $2^{<\omega} = \{f: n \to 2 \text{ where } n \in \omega\}$

For any $s \in 2^{<\omega}$ by $[s]$ denote the base open set detemined by s: $[s] = \{x \in 2^\omega : s \subseteq x\}$. Let Perf stand for the family of all perfect subsets of the space 2^ω. Recall that a proper collection of subsets of 2^ω: $\mathcal{I} \subseteq P(2^\omega)$ is called a σ - ideal iff it is closed under taking subsets and countable sums. Throughout the paper we assume that every σ - ideal $\mathcal{I}$ contains all singletons: $\forall_{x\in X}\{x\} \in \mathcal{I}$.

Let $\mathcal{I} \subseteq P(2^\omega)$ be a σ - ideal. Define the following cardinal numbers:

Definition 1. $cov(\mathcal{I}) = \min\{|\mathcal{A}| : \mathcal{A} \subseteq \mathcal{I} \wedge \bigcup \mathcal{A} = 2^\omega\}$
and

$$cof(\mathcal{I}) = \min\{|\mathcal{A}| : \mathcal{A} \subseteq \mathcal{I} \wedge \forall_{Z\in\mathcal{I}}\exists_{A\in\mathcal{A}} Z \subseteq A\}.$$

Notice that we always have $cov(\mathcal{I}) \leq cof(\mathcal{I})$.

We assume that the reader is familiar with basic concept of arithmetic of cardinal numbers. In particular, we need the notion of cofinality; recall that an uncountable cardinal number κ is called *regular* iff $\mathrm{cf}(\kappa) = \kappa$.

By $Hom(X)$ we denote the group of all homeomorphisms of the topological space X. We always assume that G is a fixed subgroup of $\mathrm{Hom}(2^\omega)$.

The following additional terminology will be useful in our proof.

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For an arbitrary $g \in G$ and $Q \in \mathit{Perf}$ we often abbreviate the image $g(Q) = \{gx : x \in Q\}$ and write simply gQ. Also for any $t \in 2^\omega$ and $A \subseteq 2^\omega$ we write $A + t = \{x + t \colon x \in A\}$.

We denote by $\mathcal{M}(P)$ the collection of all first category sets on P, where $P \in Perf(X)$.

We use a letter $\mathcal{N}$ to denote the sigma ideal of Lebesgue measure zero sets of 2^ω.

We denote by $\mathit{Trans}(2^\omega)$ the subgroup of all translations of 2^ω.

2. Introduction

Let us start with the following, classical definition:

Definition 2. A subset $S \subseteq 2^\omega$ is a *Sierpiński set* if, and only if, it is uncountable and has countable intersection with any set of measure zero.

Notice that under the assumption of Continuum Hypothesis there exists a Sierpiński set (see [9]) and, on the other hand, it is consistent that there is no Sierpiński set.

A special variation of the notion of a Sierpiński set is a κ - Sierpiński set with respect to the σ-ideal $\mathcal{I}$, namely:

Definition 3. Suppose that κ is a cardinal number and $\mathcal{I} \subseteq P(2^\omega)$ a σ - ideal. A set $X \subseteq 2^\omega$ is called a κ - Sierpiński set X with respect to $\mathcal{I}$ iff $|X| = \kappa$ and $\forall_{A\in\mathcal{I}} |A \cap X| < \kappa$.

Notice that if $\mathcal{T}$ is a σ-ideal (which contains singletons) and $\kappa = \mathit{cof}(\mathcal{I}) = \mathit{cov}(\mathcal{I})$ then there exists a κ - Sierpiński set X with respect to $\mathcal{I}$.

Recall the classical definition of perfectly meager sets (called also always of the first category sets):

Definition 4. A set X of 2^ω is a perfectly meager (AFC) set iff for every $P \in \mathrm{Perf}$, $X \cap P$ is a first category set in the topology of P.

The following notion of sets was first defined in [5] and then it has been studied most extensively in papers [6] and [7].

Definition 5. A set $X \subseteq 2^\omega$ is an AFC$'$–set if for each perfect set P there exists an F_σ–set F such that $X \subseteq F$ and for each $t \in 2^\omega$, $(F + t) \cap P$ is a first category set in the topology of P.

Notice that the notion AFC$'$ is a strengthening of the classical perfectly meager sets.

The following notion was first defined by Karel Prikry: (see [3], introduction):

Definition 6. A set $X \subseteq 2^\omega$ is called strongly meager (SFC) iff for every measure zero set $A \subseteq 2^\omega$, there exists $t \in 2^\omega$, such that $(X + t) \cap A = \emptyset$.

Notice that K. Prikry conjectured that the collection of strongly meager sets form a σ-ideal but it turned out that it is consistent that strongly meager sets are exactly the countable sets (see [3]) and that it is consistent that even the sum of two strongly meager sets need not be strongly meager set (see [2]).

It is known (see for example [5] and [7]), that $\mathrm{AFC}' \subseteq \mathrm{AFC}$ and every strongly meager set is an AFC' set.

It is also known (see [8]) that every Sierpiński set is strongly meager.

We can summarize all these inclusions in Fig. 1

$$\text{Sierpiński set} \longrightarrow \mathrm{SFC} \longrightarrow \mathrm{AFC}' \longrightarrow \mathrm{AFC}$$

Figure 1. Basic relations.

Let us define the main notion of this article.

The AFC'_G - sets

Suppose that G is a subgroup of $\mathrm{Hom}(2^\omega)$ and let X be an arbitrary subset of 2^ω.

Definition 7. Suppose that $X \subseteq 2^\omega$. We write $X \in \mathrm{AFC}'_G$ iff for every $Q \in \mathit{Perf}$ there exists $F \supseteq X$, $F \in \mathsf{F}_\sigma$ such that $\forall_{g\in G} gQ \cap F \in \mathcal{M}(gQ)$.

This notion is a natural generalization of the notion of AFC' sets.

Remarks:

It is obvious that

$$\mathrm{AFC}'_{\mathit{Trans}(2^\omega)} = \mathrm{AFC}', \quad \mathrm{AFC}'_{\{id\}} = \mathrm{AFC}, \quad \mathrm{AFC}'_{\mathrm{Hom}(2^\omega)} = [2^\omega]^{\leq\omega}.$$

It is also evident that if $G_1 \subseteq G_2$, then $\mathrm{AFC}'_{G_1} \supseteq \mathrm{AFC}'_{G_2}$.

All inclusions are summarized in Fig. 2 (where arrows denote inclusions).

$$\begin{tikzcd}
\mathrm{AFC}'_{\mathrm{Hom}(2^\omega)} \arrow[r] \arrow[d, equal] & \mathrm{AFC}'_{\mathit{Trans}(2^\omega)} \arrow[r] \arrow[d, equal] & \mathrm{AFC}'_{\{id\}} \arrow[d, equal] \\
{[2^\omega]^{\leq\omega}} & \mathrm{AFC}' & \mathrm{AFC}
\end{tikzcd}$$

Figure 2. Relations between various versions of perfectly meager sets.

Let us define:

Definition 8. Suppose that $\mathcal{I}$ is a σ - ideal of subsets of the space 2^ω.

We say that a group $G \leq \mathrm{Hom}(2^\omega)$ *has the* $(Em)_\mathcal{I}$ *property* iff there exists a perfect set $Q \in Perf$ such that for each $P \in Perf \setminus \mathcal{I}$ there exists $g \in G$ such that $P \cap gQ \notin \mathcal{M}(gQ)$.

Remarks:

One can prove that $Trans(2^\omega)$ does not have the $(Em)_\mathcal{N}$ property.

Without loss of generality we may assume that in Definition 8 P is only closed set such that $P \notin \mathcal{I}$.

We will start with the following theorem.

Theorem 1. *Let* $\mathcal{I}$ *be an arbitrary* σ *- ideal of subsets of* 2^ω *such that* $\forall_{x\in 2^\omega}\{x\} \in \mathcal{I}$.

Moreover, let $G \leq \mathrm{Hom}(2^\omega)$ *be a subgroup of* $\mathrm{Hom}(2^\omega)$ *with the property* $(Em)_\mathcal{I}$.

Then we have: $\mathrm{AFC}'_G \subseteq \mathcal{I}$.

Proof. Let $X \subseteq 2^\omega$ be a set such that $X \notin \mathcal{I}$. By the definition of the notion $(Em)_\mathcal{I}$ there is a perfect set Q such that for each closed $E \notin \mathcal{I}$ we have $\exists_{g\in G} E \cap gQ \notin \mathcal{M}(gQ)$.

Let $F \subseteq 2^\omega$ be an F_σ set such that $X \subseteq F$. We have

$$F = \bigcup_{n<\omega} F_n,$$

where $cl(F_n) = F_n$, so there exists $n_0 < \omega$ such that $F_{n_0} \notin \mathcal{I}$. Now there exists $g \in G$ such that $F_{n_0} \cap gQ$ is not meager in gQ. So we conclude, that X is not an AFC'_G set. □

The implication given in Theorem 1 is reversible under some additional set theoretical assumptions. Indeed, we have the following theorem.

Theorem 2. *Let us assume like in Theorem 1 that* $\mathcal{I}$ *is an arbitrary* σ *- ideal of subsets of* 2^ω *such that* $\forall_{x\in 2^\omega}\{x\} \in \mathcal{I}$ *and* $G \leq \mathrm{Hom}(2^\omega)$ *is a subgroup of* $\mathrm{Hom}(2^\omega)$. *Moreover, assume that*

(1) $cof(\mathcal{I}) = cov(\mathcal{I}) \leq non(\mathrm{AFC}'_G)$,
(2) $\forall_{P\in Perf\setminus\mathcal{I}}\exists_{|C|\leq\omega} 2^\omega \setminus (P + C) \in \mathcal{I}$,
(3) $Trans(2^\omega) \subseteq G$.

Then the following conditions are equivalent:

(1) $\mathrm{AFC}'_G \subseteq \mathcal{I}$
(2) G *fulfills* $(Em)_\mathcal{I}$.

Proof. Theorem 1 gives us immediately the implication (2) $\Rightarrow$ (1).

Now suppose that G fulfills $\neg(Em)_\mathcal{I}$. Since $\kappa = cof(\mathcal{I}) = cov(\mathcal{I})$ and $\mathcal{I}$ contains singletons we conclude that there exists a κ-Sierpiński set X with respect to $\mathcal{I}$ (see Def. 3). Let $Q \in Perf$ be arbitrary. From the assumption

$\neg(Em)_{\mathcal{I}}$ there exists a perfect set P such that $P \notin \mathcal{I}$ and $\forall_{g\in G} gQ \cap P \in \mathcal{M}(gQ)$. Pick a countable set $C \subseteq 2^\omega$ such that $2^\omega \setminus (C+P) \in \mathcal{I}$.

We have

$$X = \big[[2^\omega \setminus (P+C)] \cap X\big] \cup \big[(P+C) \cap X\big].$$

Since $2^\omega \setminus (P+C) \in \mathcal{I}$ we obtain $|[2^\omega \setminus (P+C)] \cap X| < \kappa$. Moreover, if $c \in C$ and $g \in G$, then $hQ \cap P \in \mathcal{M}(hQ)$, where $h \in G$ is defined by $h(x) = g(x) - c$. Hence $gQ \cap (P+c) \in \mathcal{M}(gQ)$, thus $gQ \cap (P+C) \in \mathcal{M}(gQ)$ for each $g \in G$.

Since $\kappa \leq non(\mathrm{AFC}'_G)$ we obtain $[2^\omega \setminus (P+C)] \cap X \in \mathrm{AFC}'_G$, so there exists $E \in \mathsf{F}_\sigma$, $E \supseteq X \setminus (P+C)$ such that $\forall_{g\in G} gQ \cap E \in \mathcal{M}(gQ)$. Finally, define $E^* = E \cup (P+C)$. It is easy to see that $X \subseteq E^*$ and $\forall_{g\in G} gQ \cap E^* \in \mathcal{M}(gQ)$. Hence $X \in \mathrm{AFC}'_G$ and the proof is completed, since X does not belong to $\mathcal{I}$. □

Unfortunately, we don't know whether this theorem is true under weaker assumptions. Thus we think that the following question may be of some interest.

Question 3. Can we prove the equivalence from Theorem 2 under weaker assumptions?

For any $\mathcal{F} \subseteq \mathrm{Perf}$ let us define the following cardinal coefficient:

Definition 9. $Em(\mathcal{F}, G) = \min\{|\mathcal{G}| \colon \mathcal{G} \subseteq \mathrm{Perf} \wedge \forall_{P\in\mathcal{F}} \exists g \in G \exists_{Q\in\mathcal{G}} gQ \subseteq P\}$

Let us formulate a characterization of the property (Em) in terms of the coefficient $Em(\mathcal{F}, G)$.

Assume that G has the property that for each $x \in 2^\omega$ the orbit Gx is dense in 2^ω. Then the following conditions are equivalent:

(1) G fulfills $(Em)_{\mathcal{I}}$;
(2) $|Em(\mathrm{Perf} \setminus \mathcal{I}, G)| \leq \aleph_0$.

We will need the following technical lemma (folklore for the group $G = Trans(2^\omega)$):

Lemma 1. *If $G \leq Hom(2^\omega)$ is a group such that for each $x \in 2^\omega$, Gx is dense in 2^ω, then for every sequence $\langle Q_n \rangle$ of perfect subsets of 2^ω there exists a perfect $P \in \mathrm{Perf}$ such that $\forall_{n\in\omega} \exists_{g\in G} gQ_n \cap P \notin \mathcal{M}(P)$.*

Proof. Let $v_k = [(0, \dots, 0, 1)]$ (0 k times). For each k choose $x_k \in Q_k$ and $g_k \in G$ such that $g_k x_k \in V_k$. Define $P = \overline{\bigcup_{k\in\omega} g_k Q_k \cap V_k}$, then P is a perfect set and if $k \in \omega$ then $g_k Q_k \cap P \supseteq g_k Q_k \cap V_k \notin \mathcal{M}(P)$. □

Proof. (1) $\to$ (2)
Assume that G has the $(Em)_{\mathcal{I}}$ property, i.e. there exists $Q \in \mathrm{Perf}$ such that $\forall_{P\in\mathrm{Perf}\setminus\mathcal{I}} \exists_{g\in G} P \cap gQ \notin \mathcal{M}(gQ)$. Let us define perfect sets: $\mathcal{G} = \{Q \cap [s] \colon Q \cap [s] \neq \emptyset \wedge s \in 2^{<\omega}\}$. Then $|\mathcal{G}| \leq \aleph_0$ and if $P \in \mathrm{Perf} \setminus \mathcal{I}$ then there exists $g \in G$ such that $P \cap gQ \notin \mathcal{M}(gQ)$, so $P \cap gQ \supseteq W \cap gQ \neq \emptyset$ for

some open set W. Then $g^{-1}[W] \cap Q \not\emptyset$ so there exists $Q_1 \in \mathcal{G}$ such that $Q_1 \subseteq g^{-1}[W] \cap Q$. Hence $g[Q_1] \subseteq W \cap g[Q] \subseteq P \cap g[Q]$. This proves (2).

(2) $\to$ (1). □

Next we give an useful characterization of the property $(Em)_{\mathcal{N}}$.

Theorem 4. *Let G be a subgroup of* $\mathrm{Hom}(2^\omega)$ *which contains the subgroup* $Trans(2^\omega)$*. The following two conditions are equivalent:*

(1) $\neg(Em)_{\mathcal{N}}$,

(2) *For every $Q \in Perf$ and for every $\epsilon > 0$ there exists an open set U, such that $\mu(U) < \epsilon$ and $\forall_{g\in G} gQ \cap U \neq \emptyset$*

Proof. (1) $\Rightarrow$ (2)

Assume that $\forall_{Q\in Perf} \exists_{\substack{P\in Perf\\ \mu(P)>0}} \forall_{g\in G} gQ \cap P \in \mathcal{M}(gQ)$

Let $Q \in Perf$ be any perfect set and let $\epsilon > 0$. Pick a perfect set P, $\mu(P) > 0$ such that $\forall_{g\in G} gQ \cap P \in \mathcal{M}(gQ)$. We can find finite $C \subseteq 2^\omega$ such that $\mu(2^\omega \setminus (C + P)) < \epsilon$. Now put $U = 2^\omega \setminus (C + P)$.

By way of contradiction suppose that there exists $g \in G$ such that $gQ \cap U = \emptyset$. Then $gQ \subseteq C + P$, hence there exists $c_0 \in C$ and an open set I such that $\emptyset \neq I \cap gQ \subseteq P + c_0$. Define $h(x) = g(x) - c_0$, obviously $h \in G$. Next, $hQ = gQ - c_0$ thus $\emptyset \neq hQ \cap (I - c_0) \subseteq P$, which is a contradiction with $hQ \cap P \in \mathcal{M}(hQ)$.

(2) $\Rightarrow$ (1)

Assume (2). Let R be any perfect set. Let $\{I_m\}_{m<\omega}$ be an enumeration of all basic clopen sets of 2^ω. Let

$$\epsilon_m = \frac{1}{2^{m+2}}.$$

For any $m < \omega$ we choose, using the assumption (2), an open set U_m such that

$$\forall_{g\in G} R \cap I_m \neq \emptyset \Rightarrow U_m \cap g(R \cap I_m) \neq \emptyset$$

and $\mu(U_m) < \epsilon_m$. This can be done, since $I_m \cap R$ is a perfect or an empty set.

Now put

$$U = \bigcup_{m<\omega} U_m.$$

We see that

$$\mu(U) \leq \sum_{m<\omega} \frac{1}{2^{m+2}} \leq 2 \cdot \frac{1}{4} < 1.$$

Define $F = 2^\omega \setminus U$, then we have $\mu(F) > 0$ so choose a perfect $P \subseteq F$ of positive measure.

Let $g \in G$ and I_{m_0} be given such that $R \cap I_{m_0} \neq \emptyset$.

Now $U_{m_0} \cap g(R \cap I_{m_0}) \neq \emptyset$. Moreover, since $U_{m_0} \cap P = \emptyset$ we obtain that $g(R \cap I_{m_0}) \not\subseteq P$. This means that (1) holds. $\square$

Notice that in the proof of implication (2)⇒(1) we did not use the assumption that $Trans(2^\omega) \leq G$.

In the next part we will prove theorems about relations between AFC'_G and different classes of peculiar small sets of the real line.

Theorem 5. *Assume that G is a subgroup of* $\mathrm{Hom}(2^\omega)$ *which contains* $Trans(2^\omega)$*. If G fulfills $\neg(Em)_{\mathcal{N}}$, then every strongly meager set is an* AFC'_G *set.*

Proof. Let X be a strongly meager set and let Q be an arbitrary perfect set. Since $\neg(Em)_{\mathcal{N}}$ we obtain that there exists a perfect set P such that $\mu(P) > 0$ and $\forall_{g \in G} g(Q) \cap P \in \mathcal{M}(g(Q))$. Let $C \subseteq 2^\omega$ be a countable set such that $2^\omega \setminus (P + C) \in \mathcal{N}$. Then there exists x_0 such that $(x_0 + X) \cap [2^\omega \setminus (P + C)] = \emptyset$, so $x_0 + X \subseteq P + C$, hence $X \subseteq P + C - x_0$. Let $g \in G$ be an arbitrary and let $c \in C$. Define $h \in G$ by $h(x) = g(x) - c + x_0$. Then $h(Q) \cap P \in \mathcal{M}(h(Q))$, hence $\big(g(Q) - c + x_0\big) \cap P \in \mathcal{M}(g(Q) - c + x_0)$, thus $g(Q) \cap (P + c - x_0) \in \mathcal{M}(g(Q))$. Since $c \in C$ was taken arbitrary, we conclude that $g(Q) \cap (P + C - x_0) \in \mathcal{M}(g(Q))$. This implies that $X \in \mathrm{AFC}'_G$, since $P + C - x_0 \in \mathsf{F}_\sigma$. $\square$

Remark:

This implication is reversible under CH. Namely:

Theorem 6. *Suppose that $G \leq$* $\mathrm{Hom}(2^\omega)$ *and assume that G has the $(Em)_{\mathcal{N}}$ property. Moreover, assume CH. Then there exists a strongly meager set $X \subseteq 2^\omega$ such that $X \notin$* AFC'_G*.*

Proof. Let $X \subseteq 2^\omega$ be arbitrary Sierpiński set. Then X is strongly meager ([8]). From the $(Em)_{\mathcal{N}}$ property we obtain that there exists $Q \in Perf$ such that

$$\forall_{P \in Perf \setminus \mathcal{N}} \exists_{g \in G} P \cap g(Q) \notin \mathcal{M}(gQ).$$

Suppose that E is an F_σ–set such that $X \subseteq E$. Since $X \notin \mathcal{N}$ it follows that $E \notin \mathcal{N}$. Hence there exists $P \in Perf \setminus \mathcal{N}$ such that $P \subseteq E$

Therefore $\exists_{g \in G} P \cap g(Q) \notin \mathcal{M}(gQ)$, hence $E \cap g(Q) \notin \mathcal{M}(g(Q))$. This yields $X \notin \mathrm{AFC}'_G$, which finishes the proof. $\square$

Corollary 1. *Assume that $cov(\mathcal{N}) = cof(\mathcal{N})$ and $cov(\mathcal{N})$ is a regular cardinal. Let $G \leq$* $\mathrm{Hom}(2^\omega)$ *and suppose that $Trans(2^\omega) \leq G$. Then the following conditions are equivalent:*

1. *G has the $(Em)_{\mathcal{N}}$ property.*
2. $\mathrm{AFC}'_G \subseteq \mathcal{N}$.

Proof. The implication (1) $\Rightarrow$ (2) follows immediately from Theorem 1. Assume $\neg(Em)_{\mathcal{N}}$. Since $cov(\mathcal{N}) = cof(\mathcal{N})$, there exists a $cof(\mathcal{N})$ – Sierpiński set. By Lemma 8.5.4 from [1] if there exists a κ – Sierpiński set and $\mathrm{cf}(\kappa) = \kappa > \omega$, then every set of size $< \kappa$ is strongly meager. Hence by Theorem 5 we conclude that $non(\mathrm{AFC}'_G) \geq cof(\mathcal{N})$ thus all assumptions of Theorem 2 are satisfied. □

REFERENCES

[1] T. Bartoszyński, H. Judah, *Set Theory: on the strucure of the real line*, A. K. Peters, Wellesley, Mass., 1995.

[2] T. Bartoszyński, S. Shelah, *Strongly meager sets do not form an ideal*, Journal of Mathematical Logic, 1 (1) (2001), 1–34.

[3] T.J. Carlson, *Strong measure zero and strongly meager sets*, Proceedings of The American Mathematical Society, 118 (2) (1993), 577–586.

[4] A. Nowik, *Remarks about transitive version of perfectly meager sets*, Real Analysis Exchange, 22 (1) (1996/97), 406–412.

[5] A. Nowik, M. Scheepers, T. Weiss, *The algebraic sum of sets of real numbers with strong measure zero sets*, Journal of Symbolic Logic, 63 (1) (1998), 301–324.

[6] A. Nowik, T. Weiss, *Not every Q-set is perfectly meager in the transitive sense*, Proceedings of The American Mathematical Society, 128 (10) (2000), 3017–3024.

[7] A. Nowik, T. Weiss, *The algebraic sum of a set of strong measure zero and a perfectly meager set revisited*, East-West Journal of Mathematics, 2 (2) (2000), 191–194.

[8] J. Pawlikowski, *All Sierpiński sets are strongly meager*, Archive for Mathematical Logic, 35 (1996) 281–285.

[9] W. Sierpiński, *Sur l'hypothèse du continu ($2^{\aleph_0} = \aleph_1$)*, Fundamenta Mathematicae, 5 (1) (1924), 177–187.

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